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The Summer Institute in Mathematics for Undergraduates (SIMU) at the University of Puerto Rico – Humacao

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1. Introduction

The Summer Institute in Mathematics for Undergraduates (SIMU) at the University of Puerto Rico – Humacao is a six-week research program in the mathematical sciences. Professors Ivelisse Rubio and Pablo V. Negrón from the University of Puerto Rico - Humacao, and Herbert A. Medina from Loyola Marymount University served as Co-Directors of the program in 1998 and 1999. Twenty-seven students participated in SIMU in 1998 and twenty-four participated in 1999.

The program is designed for talented Chicano/Latino and Native Americans[1] undergraduates who want to engage in undergraduate mathematics research. The objectives of SIMU are:

1. to provide SIMU participants a rich and intensive research experience in the mathematical sciences—an experience that has proven successful in getting Chicano/Latino and Native American undergraduates to pursue graduate studies in the mathematical sciences;
2. to familiarize SIMU students with research protocols and techniques, with collaboration between peers, and with utilizing faculty as effective resources while conducting research—skills that

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[1]The populations served by the Society for Advancement of Chicanos and Native Americans in Science (SACNAS), an organization with which the program has close ties.

will help them succeed in their undergraduate and graduate education and their research careers;
3. to create an undergraduate research community that stimulates students to reach their fullest potential.
4. to offer SIMU participants workshops that will help them to develop skills and techniques needed in research careers in the mathematical sciences;
5. to build a network of researchers and peers that, through mentoring and collaboration, will help SIMU participants to continue their research activity and excel in their graduate education;
6. to introduce SIMU students to successful Chicano/Latino and Native American researchers and graduate students so as to encourage and motivate them to pursue careers in mathematics;
7. to provide SIMU participants the opportunity to attend undergraduate research forums where they will present the research they completed during SIMU;
8. to offer SIMU participants workshops that will teach them skills and techniques that will maximize their likelihood of admission to graduate programs best suited to their needs as well as their likelihood of securing financial support for such programs;
9. to enhance the overall academic portfolio of SIMU students by providing them the opportunity to work with and meet leading mathematicians so that their future applications to graduate school and fellowships are strengthened;
10. to monitor the educational progress and research activity of SIMU participants for at least five years after their participation in the institute, such monitoring being a measure of the program's success.

These objectives will contribute significantly towards the goal of SIMU: *to increase the number of Chicanos/Latinos and Native Americans earning graduate degrees and pursuing careers in the mathematical sciences.*

The reason for working to increase the number of Chicanos/Latinos and Native Americans pursuing research careers in the mathematical sciences can be summarized as follows. Historically, Chicanos/Latinos and Native Americans have been severely underrepresented in mathematics and science. This pattern continues today. For example, between 1989 and 1996, only 3.35% of all bachelor's degrees in the mathematical sciences were awarded to Hispanics and Native Americans in the U.S. and U.S. territories. The percentages for master's degrees during the same period is 1.78%; and only 1.15% of all doctorate degrees in the mathematical sciences between 1988 and 1997 were awarded

to Hispanics/Latinos and Native Americans [**3, 4**]. These numbers are alarmingly low when one considers that these two ethnic groups account for 12.2% of the current U.S. population and that the U.S. Census Bureau projects that they will account for 18.6% of the U.S. population in 2025 [**5, 6**]. The leading position in mathematics and science that the U.S. occupies will be maintained only if all groups in our society actively participate in these fields. Indeed, if, as predicted by the U.S. Census Bureau, Latinos and Native Americans make up 18.6% of the U.S. population in the year 2025, and continue to account for only 1.15% of all doctorates in mathematics, then the U.S. will be leaving untapped a resource of millions of people from which to produce professional mathematicians; this puts the U.S. at a huge disadvantage to countries who tap their population effectively for mathematicians and scientists. One can ask the question, can a country that essentially does not use one fifth of its population to produce mathematicians continue to be a world leader in this field? We believe it is not possible, and SIMU's goal is to address this important issue.

2. Funding Information

The National Security Agency (NSA) provided the largest grants for the 1998 and 1999 SIMU's. The University of Puerto Rico has provided funds for SIMU in 1998, 1999 and will do so again in 2000. The National Science Foundation (NSF) provided significant funding for SIMU in 1999. The Alliance for Minority Participation (AMP) has funded the stipends and some travel to conferences for Puerto Rican students in both the 1998 and 1999 SIMU's. The National Aeronautics and Space Administration (NASA), through the Puerto Rico Space Grant program also provided funding for the 1998 and 1999 SIMU's. Finally, the Society for Advancement of Chicanos and Native Americans in Science (SACNAS) has paid expenses for three colloquium speakers during the past two years, and, after selecting them through a competitive process, has covered travel expenses of all SIMU students attending the 1998 and 1999 SACNAS Conferences.

3. Program Philosophy and Structure

The Co-Director's undergraduate research philosophy can be summarized as follows.

1. Create an intense and collaborative academic and intellectual environment by ensuring that students understand the high quality

and quantity of work that is expected of them and become self-motivated to try to meet these standards and perhaps set higher ones for themselves;
2. Create a structure that maximizes faculty-to-student mentoring and support in the early stages of the program and that makes student-to-student interaction the primary dynamic while students are conducting mathematical research under the direction of their faculty mentor;
3. Immerse students in an intensive training that will give them all the necessary mathematical and computational knowledge and tools so that they successfully can engage in quality research; and
4. Engage students in challenging, interesting and accessible mathematical topics and research projects that results in a rich research experience.

SIMU is structured so that in addition to the Co-Directors, each summer two faculty direct the research of students participating in the program. The selection of these faculty, called "seminar leaders," is done by the Co-Directors on a year-to-year basis so as to allow a variety of mathematical fields to be the research areas of the program. The seminar leaders are mathematicians who are experts in their field of research, who have demonstrated excellence in and are committed to undergraduate education, and who share the vision and philosophy of SIMU. Each year, the seminar leaders are assisted by four seminar associates, two for each seminar. The organizers, in consultation with the seminar leaders, recruit the seminar associates. In the future, if possible, the seminar associates will come from the pool of former SIMU students who are in graduate programs in the mathematical sciences.

SIMU is designed so that the students are immersed in their research topic from the first day. Each day during the first three weeks, there are four different organized day activities and study groups at night which create an intense working environment that sets the pace and creates the conditions for the research that the students conduct during the institute. The pre-research seminar, working sessions, computational laboratory, and intense academic environment give students sufficient background so that after three weeks they can conduct an undergraduate-research project. That is, by the end of the third week, students are "undergraduate experts" in the mathematical field in which they will conduct research.

3.1. Pre-Research Seminars and Working Sessions. The pre-research seminar is a three-week intense training that provides the necessary background and prepares students to do quality research in a field of the mathematical sciences that is accessible to undergraduates. Each student participates in one of two pre-research seminars.

The pre-research seminars meet daily Monday – Thursday during the first three weeks of the institute for two seventy-five minute, morning sessions. The first daily session is devoted to the presentation of new material in an interactive lecture setting. The content of the seminars is designed to familiarize students with the fundamental concepts of the mathematical topic. During the second daily session, students work in groups on problems designed by the seminar leaders. The problems are designed to promote and enhance the material covered in the interactive lecture and to familiarize students with the different research projects. During these sessions, the seminar leader and associates float and help students to tackle the problems.

During the afternoons of this pre-research phase, students have a ninety-minute study session, where they are assisted by the seminar leader and associates. Also, seminar leaders and associates hold consultation sessions in the evening in the student residences. These sessions allow students to engage in at least three more hours of daily structured work. This proved in the 1998 and 1999 SIMU's to be a key in creating and sustaining a supportive and intensive academic and research environment. During all of these structured study sessions, students are encouraged to use their peers as the primary resource for answering questions.

3.2. Computational Laboratory. A computational laboratory meets Monday – Thursday in the afternoon for ninety minutes. It is dedicated to activities that supplement the mathematics from the pre-research seminar and prepare students to tackle the research project. The seminar leaders and associates design laboratories that give students the opportunity to use packages like Maple, Mathematica, Matlab, and Splus to solve problems, develop and test conjectures, and learn the value of the computer as a tool in the mathematical sciences. By the end of the third week of the institute, students are familiar with software packages and computational techniques that expose them further to the tools in use by mathematicians who work in the field.

After the first three weeks, students are ready to devote all their time to work in groups on their research projects.

3.3. Research Projects. By the end of the second week, the seminar leader has distributed a description of research projects. Students

begin to do preliminary reading and literature searches on the projects immediately. By the end of the third week, with the aid of their seminar leader, students have selected a research project, organized themselves and designed a plan of attack to tackle the project. At this juncture, each team of students makes a presentation to SIMU participants in their seminar in which they give an overview of their research project and the methods and techniques that they hope to use to tackle it. These presentations mark the end of the pre-research phase of SIMU.

For the remainder of the program, the seminar leader and associates are available to meet with students during the day and night if necessary. Each research team gives a daily progress report to the seminar leader or one of the associates. In addition, the Co-Directors are available during the day and night if students want to discuss their project with someone other than the seminar leader and research associates.

At the end of the program, each team of students makes an oral presentation to all SIMU students and faculty at an end-of-program symposium. During the 1998 and 1999 SIMU's, faculty from the UPR – Humacao, other UPR campuses, and mathematicians from the NSA and NSF have been present for the end-of-program symposium. Each research team also writes a technical report describing the results of their research. These technical reports are published each year by SIMU [**1, 2**].

The research topics during the first two years of SIMU were Gröbner bases, computational number theory, and probability and statistics. Computational algebra, particle methods in fluid dynamics, Gröbner bases, coding theory, and wavelets will be among the research topics during the next three years.

3.4. Other SIMU Activities: Colloquia, Workshops and Recreational Activities. Speakers from across the U. S. and Puerto Rico are invited to SIMU to give a colloquium talk. The aim is not only to give students the opportunity to hear a talk on current research, but also to provide students with another role model and future mentor in the mathematical sciences. Female and male colloquium speakers are chosen to represent a broad spectrum of mathematical disciplines as well as ethnic and cultural backgrounds, including those similar to the students' backgrounds.

There are five colloquium speakers, one each during the first five weeks of the institute. Colloquium speakers arrive in Humacao on Thursday and stay until Sunday. Each speaker gives a one-hour colloquium talk on Friday afternoon. In addition to giving their talk, colloquium speakers are asked to attend the institute's seminars and

laboratories, to interact with students on an informal basis, to attend a recreational/cultural outing on Saturday, and to discuss graduate programs at their institution or career options at their laboratory, government agency or corporation.

SIMU has a collection of workshops designed to assist students in the pursuit of a graduate education and the development of skills that are important to mathematicians.

Dr. Colette Patt, Director of Diversity Programs in the Physical Sciences at the University of California, Berkeley, has given workshops to inform students about the graduate education options available to them, and the funding possibilities available for attending graduate school. Dr. Patt addresses questions/issues such as the significant differences between a master's and a doctoral program, the funding opportunities available for most graduate programs, and the benefits of obtaining a graduate degree. In addition to this basic information, Dr. Patt also presents successful techniques for applying to graduate school. She discusses the elements that constitute a good statement of purpose, the types of professors from whom one should seek letters of recommendation, and successful techniques for addressing not-so-stellar semesters. In addition to the workshop, Dr. Patt meets individually with students to develop a blueprint for applying to graduate school. For example, each student receives individual academic counseling to help him/her develop a list of universities to which s/he hopes to apply, a list of fellowships to which s/he should apply, a list of faculty whom s/he will ask for letters of recommendation, and a cover letter to accompany requests for letters of recommendation.

The other three workshops are led by the Co-Directors and are devoted to the development of skills that are important to every mathematician. The first is devoted to learning LaTeX, the typesetting program most widely used by mathematicians. The second workshop familiarizes students with good practices in preparing and delivering a mathematics oral presentation. The third instructs students on successful techniques in preparing a mathematics poster for a conference.

The working environment in SIMU is very intense. During the first three weeks from Monday to Thursday students work on a mathematical topic in a structured setting for more than eight hours per day and many more hours on their own. During the second three weeks, students work almost continuously on their project. Creating and sustaining an academic and research environment in which students are challenged, mentored, and supported by faculty and peers is one of the biggest challenges and one of the key components of undergraduate

summer institutes. To sustain this environment, it is important that students also engage in non-academic activities.

In the afternoons, students have time to go to the university gym and to the swimming pool. SIMU organizes a couple of athletic competitions during the first and fifth weeks of the institute. Also, on Saturdays, SIMU organizes outings to places like historic Old San Juan, the capital of Puerto Rico; El Yunque, the only tropical rain forest among the National Parks of the U.S.; and the Arecibo Observatory.

4. Results of 1998 and 1999 SIMU's

Almost all SIMU students have communicated orally and in writing in post-program questionnaires that the program was an excellent experience. Here are some quotes from SIMU students.

- *SIMU helped to convince me that one of the things to which I want to devote my life is to mathematical research. It was an excellent experience.*
  Omar Colón-Reyes, University of Puerto Rico – Humacao
- *SIMU's main purpose was to encourage us to work hard and to our fullest potential, however I feel that it did a lot more than that. It taught me to motivate myself. I encourage others to participate in the program.*
  Carina Nieves, Kean University
- *What I love the most about SIMU is the attention that we get from the faculty. The level of interaction between students, TA's and professors is the best! SIMU has made me come to the conclusion that I can, and will, succeed in graduate studies in mathematics.*
  Omar Zuñiga, University of California, Riverside.

Most SIMU students continue to be members of the academic and research communities built during their time in Humacao. Indeed, several 1998 SIMU students were in contact with each other during their graduate school application process. A student from each of the past two SIMU's has created an Internet club for the SIMU students from that year.

To sum, the Co-Directors witnessed the transformations that students experienced in the 1998 and 1999 SIMU when they realized their fascination with and passion for the mathematics that they were learning and the research they were conducting. These types of transformations are often the motivating factor in inspiring a student to pursue a graduate degree, and the 1998 and 1999 SIMU were successful in achieving them.

4.1. Quantitative Results from the 1998 SIMU. Twenty-seven students, sixteen men and eleven women, from sixteen universities participated in the 1998 SIMU. The fifteen students who participated in the Number Theory Seminar under the direction of Professor Carlos Moreno, City University of New York, worked on individual research projects; the twelve students who participated in the Gröbner basis seminar under the direction of Professor John B. Little, College of the Holy Cross, worked on three group research projects. Each student or group of students gave a talk at the end-of-program colloquium series and produced a technical report published by SIMU [**1**]. Twenty-five of the twenty-seven students presented posters on their research at the 1998 SACNAS Conference in Washington, D.C. in October, 1998. Thirteen SIMU students presented nine posters at a session sponsored by the Mathematics Association of America (MAA) at the Joint Mathematics Meetings in San Antonio in January 1999. At both conferences, one of the SIMU posters won an award. Many other students presented posters and gave talks based on their SIMU research at other conferences (e.g., Southern California Conference on Undergraduate Research, Nebraska Conference for Undergraduate Women in Mathematics, Junior Technical Meeting in Puerto Rico) and their universities. Over seventy percent of these students had not participated in undergraduate research prior to SIMU.

The success of these students in continuing their graduate education is also impressive. The Co-Directors gathered data on these students in a post-SIMU May 1999 questionnaire. A summary of the information gathered therein is as follows.

1. The Co-Directors received twenty-three questionnaires from a possible twenty-seven. (The Co-Directors are trying to get the other four students to return their questionnaires.)
2. Of the fourteen respondents who graduated on or before Spring 1999, ten (71%) applied and were accepted to graduate programs. One of these students has postponed graduate school for a year, and the rest will be enrolled in a graduate program in Fall 1999.[2]
3. One of these students won a Ford Foundation Fellowship.
4. All respondents who graduated in Spring 1999, have applied or said that they would apply to graduate school in the future.
5. All of the students who began graduate school in Fall 1999 stated that SIMU helped them, by motivating and providing them with

[2]The universities that these students are enrolled are Cornell University, New Mexico State University, Stanford University, University of Arizona, University of Colorado, University of Iowa, University of Maryland, and University of PuertoRico – Mayagüez.

important information, in their application process to graduate school.

4.2. Quantitative Results from the 1999 SIMU. Twenty-four students, twelve men and twelve women, from eighteen universities participated in the 1999 SIMU. They worked on eight research projects in groups of three. Four projects in Gröbner basis were directed by Professor John B. Little, and four projects in probability and statistics were directed by Professor Rudy Guerra, Southern Methodist University. All eight groups gave talks at the end-of-program symposium and wrote technical reports which have be published by SIMU [**2**]. Twenty-two students will present eight posters on their SIMU research at the SACNAS Conference in Portland, OR in October, 1999. The Co-Directors expect that at least a dozen students will present a poster in the MAA poster session at the Joint Mathematics Meetings in Washington, D.C. in January 2000.

Information gathered by the Co-Directors in the end-of-program student questionnaire shows that seventeen of the twenty-four 1999 SIMU students had not worked on mathematical research prior to SIMU; that all students plan to continue to engage in research after SIMU; and that SIMU has either "increased" or "increased significantly" the desire of nineteen students to pursue a graduate degree in mathematics or science.

5. The Future of SIMU

Professors Rubio and Medina will serve as SIMU Co-Directors in the future. They envision running SIMU for several years with twenty-four students each year. They will continue to apply yearly for funding to the NSA, and have applied for funding to NSF to establish SIMU as a Research Experience for Undergraduates (REU) site for 2000 – 2002.

The seminar leaders and research topics during the next few years include Reinhard Laubenbacher, New Mexico State University, *Computaional Algebra*; Ricardo Cortez, Tulane University, *Particle Methods in Fluid Dynamics*; John B. Little, College of the Holy Cross, *Gröbner bases* and *Coding theory*; and Herbert A. Medina, *Orthogonal expansions and wavelets.*

In addition to planning the next few SIMU's, the Co-Directors are building partnerships with faculty at schools that have graduate programs to facilitate admissions of SIMU students to these programs and to strengthen SIMU.

The Mathematics Department of Cornell University has included SIMU as part of its NSF Vertical Integration of Research and Education

in the Mathematical Sciences (VIGRE) grant proposal. If the proposal is funded, it would provide funding for 1. a Cornell mathematics faculty to travel to SIMU to give a colloquium talk every year; 2. a Cornell graduate student to serve as a SIMU seminar associate every year; 3. one year of Cornell support and two years of VIGRE support for SIMU students admitted to that department's graduate program. Cornell also would waive the application fee for SIMU students applying to its graduate mathematics program.

Ricardo Cortez has included SIMU as part of his NSF CAREER grant proposal. Dr. Cortez will serve as a SIMU Seminar Leader in 2002, and if his CAREER proposal is funded, SIMU students would be able to participate in research at Tulane; and Ricardo Cortez's salary during SIMU 2002 would come from his CAREER grant. Both of these activities would facilitate the application and admission process of students to the graduate mathematics program at Tulane.

Professor Laubenbacher is writing a proposal to the Alfred P. Sloan Foundation requesting graduate fellowships for SIMU students. If this proposal is funded, it will provide fellowships for SIMU students accepted to the graduate program in mathematics at NMSU. Two 1998 SIMU students and a 1999 SIMU seminar associate are currently enrolled in the mathematics graduate program at NMSU and the availability of fellowships would strengthen further the SIMU-to-NMSU pipeline.

6. Conclusion

The goal of SIMU is *to increase the number of Latinos and Native Americans earning graduate degrees and pursuing careers in the mathematical sciences*. The 1998 and 1999 SIMU's have demonstrated that the model of engaging students in quality undergraduate mathematics research and continuing to mentor them after the program is a successful one for a program with this goal. The Co-Directors plan to continue to fine tune the model so that within a few years, SIMU will have made an impact on the number of Latino and Native Americans earning graduate degrees in the mathematical sciences.

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